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T2 ________		F2 ________
T3 ________		F3 ________
T4 ________	Problem Chosen	F4 ________
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Never Worry About Your Baking

--Gourmet Gospel

Summary

People always worry about the overcooking of cakes. The round pan can distribute heat evenly but wastes the space of the oven. The rectangular pan can make full use of space but always causes the cake overcooked, especially in the corners. Then, how can we design the shape of the pan to take both factors into consideration?

At first, we calculate the temperature of every part of the pan by the two-dimensional non-steady-state heat transfer formula. Secondly, we set two standard temperatures of the roasted temperature and scorched temperature. Thirdly, use Visual Studio 2010 to make C++ programming to simulate the baking of the cake, so that we can get the distribution of heat and the scorched rate. Lastly, we use R 2.15.1 to plot the figures of heat distribution. We find that the rounded rectangle pan distribute the heat most evenly.

To maximum the number of pans fitted in the oven, we need to minimum the wasted space of the oven. Therefore, we calculate the wasted space of different shapes, and the lower wasted space is the better shape is. After making the data dimensionless, we create an evaluation function by the weight P of the factor which aims for maximizing number. Then, we plot a scatterplot of the wasted rate and scorched rate, and obtain the smallest convex hull to realize the linear programming. The result shows that rounded rectangle pan is the best, and the radius increases with the increasing of the weight P. When P=0, which means that we just aim to maximum the number, the radius=0, that means the rectangle one is the best. We choose five ovens that have different width to length ratio of W/L, and calculate the best shape for them in different weight of P. We summary the results in table5.3.1.

Finally, we compare the wasted rate and scorched rate of the best shape for nine different ovens. Then we find that the shorter the W/L is the better the shape is. Therefore, if we know the area of the oven that can fit pans, we should choose the shape that has the shortest W/L, otherwise, choose the pan whose width to length ratio equals to the oven's. At the last two pages of our solution paper is the advertisement for presenting our design of the pan.

Above all, our model can design the best shape for any different kinds of ovens, and the algorithm which is used to solve the model is stable and robust.

Contents

1. Introduction

Most families or bakery use the oven to bake delicious cakes, but there're always some cakes were overcooked, especially the one that is at the edge of the baking pan. One of the most significant causes is the shape of the pan.

Except for the edges of the pan, the cakes received from the other aspects of heat are the same. The cakes at the edges of the plates having a greater heating area, thus, it continues high temperature and is easy to be overcooked.

For rectangular pan, heat is concentrated in the 4 corners cause that the product gets overcooked. For round pan, the heat is distributed evenly over the entire outer edge and the product is not overcooked at the edges. However, since most ovens are rectangular in shape using, round pans is not efficient with respect to using the space in an oven.

We can develop a model to show the distribution of heat across the outer edge of a pan for pans of different shapes – form rectangular to circular and other shapes in between. Subsequently, establish a model that can be used to select the best shape of pan under the following conditions:

1. Maximize number of pans that can fit in the oven (N)
2. Maximize even distribution of heat (H) for the pan
3. Optimize a combination of conditions (1) and (2) where weights P and (1-P) are assigned to illustrate how the results vary with different values of *W/L* and P.

2. Assumptions

- A width to length ratio of W/L for the oven which is rectangular in shape.
- Each pan must have an area of A.
- Initially two racks in the oven evenly spaced.
- The temperature of oven is evenly distributed and unchanged
- The initial temperature of the cake is uniform.

3. Symbol description

- R_i: The wasted rate
- H_i: The scorched rate
- P_i: The weight of condition1

4. Analysis of the problem

4.1 The heat distribution

Develop a model to show the distribution of heat across the outer edge of different shapes -- rectangular to circular and other shapes in between.

According to the introduction, the cake is most likely to get overcooked at the corners of the rectangular pan. And in a round pan the heat is distributed evenly over the entire outer edge which makes the cake is not overcooked. Thus, we try to replace the corners by other shape that is sleek.

To change the shape of the pan, firstly, we replace the corner by a quarter of circle whose radius is R. With the increase of radius, the shape of pan gradually turns into a shape that has a rectangular in the middle of two semicircles. Secondly, we view the semicircle as a semiellipse whose semi-minor axis is equal to semi-major axis. With the increase of semi-major axis, the shape of pan gradually turns to be oval-shaped. Finally, we decrease the semi-major axis of the oval pan which gradually turns the shape into a round.

For any kind of shape, we need to calculate their heat distribution. We assume the temperature of oven is evenly distributed and unchanged, and the initial temperature of the cake is uniform. The temperature of every part of the cake depends on the temperature of previous time of the adjacent region and its own. To simplify the model, we discretize the area of the cake, and use their central temperature to represent the temperature of every piece. Then, we establish the heat transfer model, and use computer simulation to solve it out.

4.2 Create an ideal shape

Establish a model that can be used to select the best shape of pan under the conditions of the introduction.

To choose the shape of pan that can maximum the number of pans fitted in the oven, first we calculate the area (B) of the bounding rectangle of each shape. Since the area of pan is A, the ratio of A and B the greater the better, which means if A/B is greater than others, then that shape of pan can fit more in the oven.

To choose the shape of pan that won't make cake overcooked, we define a scorched rate as H_i. The lower the H_i is the better the shape is.

To combine the two conditions, we set P as the weight of condition1, and create evaluation function for it. Then, we can obtain the different optimal shapes for different requirements.

At last, we choose nine typical sizes of ovens, and calculate the best shape for them to explore the influence of the oven size to the shape.

5. Models

5.1 Heat transfer model

5.1.1 Preparation of the model

The heat transfer during baking cakes includes conduction and radiation. Due to the edges of the pan is connected with the cake most of the time; the main form of heat transfer is the conduction. Thus, we view the heat transfer during baking cakes as thermal conduction.

Steady state conduction is the form of conduction that happens when the temperature difference driving the conduction are constant, so that (after an equilibration time), the spatial distribution of temperatures (temperature field) in the conducting object does not change any further. Transient conduction is the mode of thermal energy flow that during any period in which temperatures are changing in time at any place within the object.

During the process of baking cake, the temperature of the different parts of the cake isn't in a continuously changing until cake is baked. Thus, the process of baking cake belongs to the transient conduction.

The basic idea of solving the heat transfer model is as follows: Replace the continuous temperature distribution within the object by the temperature approximation of a finite number of discrete points, which can turn the problem that calculates the continuous temperature distribution into a problem that calculates the temperature. Therefore, the main content of the heat transfer model is to discretize the solution domain and to establish and solve the node temperature algebraic equations.

The basic steps of solving the heat transfer model:

1. Simplify the problem reasonably and establish the physical model. Then establish a complete mathematical model to calculate the heat transfer equation and single-valued conditions.
2. Discretize the solution domain: divide the target area into some smaller subdomains (their size are the same) ， and then, view the temperature of the center point as the temperature of the subdomains.
3. Establish and solve the node temperature algebraic equations.

5.1.2 Establishment of the model

We assume the baking from the bottom of the pan is uniform, and the shape of the pan only influences the baking from the edges of the pan. Thus, we can view the problem as a two-dimensional non-steady-state heat transfer problem.

Divide the pan (the area of the pan is A) into n pieces, and the area of each piece is Δa. Meanwhile, divide the baking duration into 'm' times of intervals, and the step length is one. Then, the temperature of one of the piece can replace by the temperature of its center in different time.

Through the heat balance method, we can establish a different equation of the inner region and the boundary region's temperature.

For the two-dimensional non-steady-state heat transfer problem, the thermal equilibrium of the inner region(i, j) is expressed as follows: at time k, separately transfer into the flow of heat per unit time from the adjacent region $(i-1, j)$, $(i+1, j), (i, j-1), (i, j+1)$ is Q_1, Q_2, Q_3, Q_4, and the sum is equal to the heat increments dU. Then we have

$$Q_1 + Q_2 + Q_3 + Q_4 = dU. \tag{5.1.1}$$

If we calculate the temperature on the time rate of change by forward differencing, then we have

$$\lambda \frac{t_{i-1,j}^k - t_{i,j}^k}{\Delta a} + \lambda \frac{t_{i+1,j}^k - t_{i,j}^k}{\Delta a} + \lambda \frac{t_{i,j-1}^k - t_{i,j}^k}{\Delta a} + \lambda \frac{t_{i,j+1}^k - t_{i,j}^k}{\Delta a} = \Delta a \rho c \frac{t_{i,j}^{k+1} - t_{i,j}^k}{1}. \tag{5.1.2}$$

Define $\alpha = \dfrac{\lambda}{\rho c}$, rearranging (5.1.2) produces

$$t_{i,j}^{k+1} - t_{i,j}^k = \alpha \frac{t_{i-1,j}^k + t_{i+1,j}^k + t_{i,j-1}^k + t_{i,j+1}^k - 4t_{i,j}^k}{(\Delta a)^2}. \tag{5.1.3}$$

Define $F_{O\Delta} = \frac{\alpha}{(\Delta a)^2}$, and $F_{o\Delta}$ is a Fourier number. Rearranging (5.1.2) produces an explicit formula:

$$t_{i,j}^{k+1} = F_{o\Delta}(t_{i-1,j}^{k} + t_{i+1,j}^{k} + t_{i,j-1}^{k} + t_{i,j+1}^{k}) + (1-4F_{o\Delta})t_{i,j}^{k}. \quad (5.1.4)$$

The formula (5.1.4) is called the two-dimensional transient conduction differential equation of the internal regional temperature. It shows that

1. The temperature of an interior region at a particular moment can be calculated by the temperature of the region and its adjacent areas in the previous moment. Thus, we needn't to calculate the simultaneous equations but to calculate by the initial temperature.
2. The stability condition of the formula (5.1.4): The temperature of each internal region in time k +1 depends on the temperature of the region and its adjacent areas in time k. Within this region, the higher the temperature is at time k, the temperature at its corresponding time k+1 is higher as well. Therefore, the coefficient of $t_{i,j}^{k}$ in formula (5.1.4) can't be negative which means $F_{o\Delta} \le \frac{1}{4}$.

As for the thermal equilibrium of the boundary region, the sum of the heat flow from the edges and the adjacent areas is equal to the heat increments. If the heat transferred from the edges of pan to the boundary region is determined and the temperature of the pan maintains equal to the oven, we can turn the problem of boundary region into the problem of inner region.

5.1.3 Model solution

Use C++ Programming to calculate the heat transfer model, and simulate the unsteady heat transfer process.

1. The initial temperature of the cake is set to 20, the initial temperature of the oven is set to 180, and the Fourier number $F_{o\Delta}$ is set to 0.22.
2. Discretize the heating area into a grid of 100×100, and put the different kind of pan in it (the area of the pan is A). Then, calculate the discrete regional grid covered by the pan.
3. According to the initial conditions, calculate the next temperature of the covered grid through formula (5.1.4).
4. Set all temperature reach 60 or more as the goal, which means we consider the baking of the cake is finished. After achieved, record the temperature of each grid.

According to the above process, we can calculate the heat distribution of different kind of pan. And we describe five typical pans as follows.

Rectangular pan

The conditions is

$$\begin{cases} \mathrm{WL} = \mathrm{A} \\ \frac{\mathrm{W}}{\mathrm{L}} = \frac{\mathrm{w}}{\mathrm{l}} \\ \mathrm{W} > 0, L > 0 \end{cases} . \quad (5.1.5)$$

Solving for (5.1.5) we obtain

$$\mathrm{W} = \sqrt{\mathrm{A}/(\frac{\mathrm{w}}{\mathrm{l}})}, \mathrm{L} = \sqrt{\mathrm{A}(\frac{\mathrm{w}}{\mathrm{l}})} . \quad (5.1.6)$$

Figure5.1.1 shows the schematic diagram of rectangular pan.

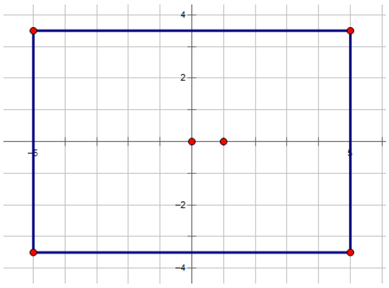


Figure 5.1.1 the schematic diagram of rectangular pan.

To show the heat distribution of the pan, we get figure5.1.2 by R2.15.1. In addition, the color to be filled is "rainbow" causes the central location's color is orange which represent a low temperature here. And the one on the right-hand side of the figure is the temperature corresponding to different colors.

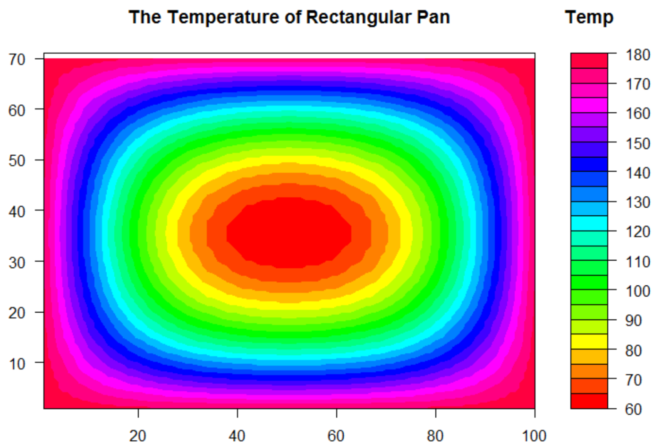


Figure 5.1.2

Rounded rectangle pan

We replace the corner of the rectangle by a quarter of circle, so that we get the rounded rectangle pan.

Set the radius of the circle as a, then it should meet

$$\begin{cases} \mathrm{WL}-(4-\pi)a^2=\mathrm{A} \\ \frac{\mathrm{W}}{\mathrm{L}}=\frac{\mathrm{w}}{\mathrm{l}} \\ \mathrm{W}>0, L>0 \end{cases}. \tag{5.1.7}$$

Solving for (5.1.7) we obtain

$$\begin{cases} \mathrm{L}=\sqrt{\frac{1}{w/l}(A+(4-\pi)a^2)} \\ \mathrm{W}=\sqrt{w/l\,(A+(4-\pi)a^2)} \end{cases}. \tag{5.1.8}$$

Figure5.1.3 shows the schematic diagram of rounded rectangle pan.

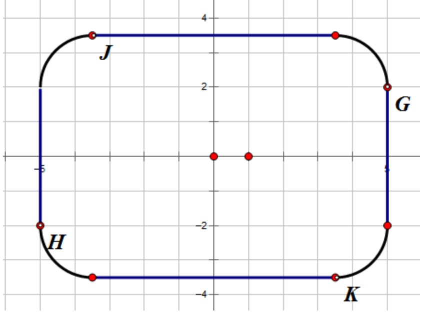


Figure 5.1.3 the schematic diagram of rounded rectangle pan.

Figure5.1.4 shows the heat distribution of the rounded rectangle pan.

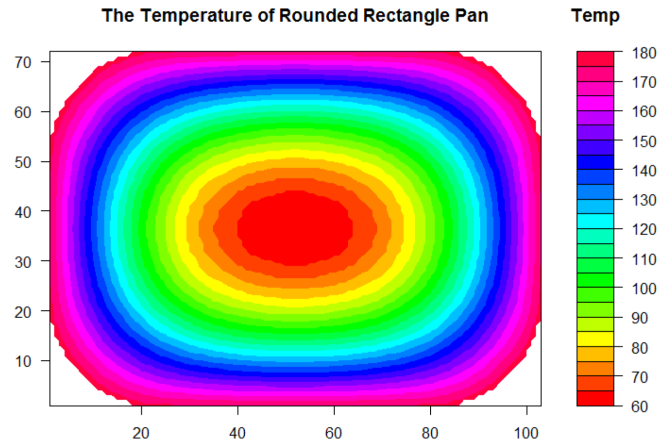


Figure 5.1.4

Oval plus rectangle pan

We add a rectangle into the oval, and then get the oval plus rectangle pan. Set the length of the semi-major axis as a, then it should meet

$$\begin{cases} \frac{\pi W a}{2} + (L - 2a)W = A \\ \frac{W}{L} = \frac{w}{l} \\ W > 0, L > 0 \end{cases} . \tag{5.1.9}$$

Solving for (5.1.9) we obtain

$$\begin{cases} L = \frac{\frac{(4-\pi)}{2}a + \sqrt{\left(\frac{(4-\pi)}{2}a\right)^2 + \frac{4A}{\frac{w}{l}}}}{2} \\ W = L\left(\frac{w}{l}\right) \end{cases} \tag{5.1.10}$$

Figure5.1.5 shows the schematic diagram of oval plus rectangle pan.

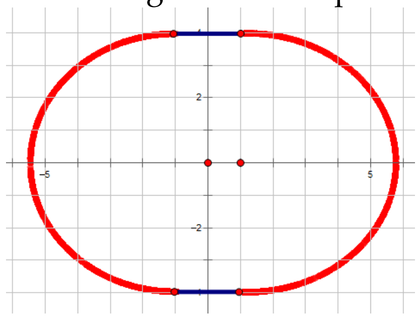


Figure 5.1.5 the schematic diagram of oval plus rectangle pan.

Figure5.1.6 shows the heat distribution of the oval plus rectangle pan.

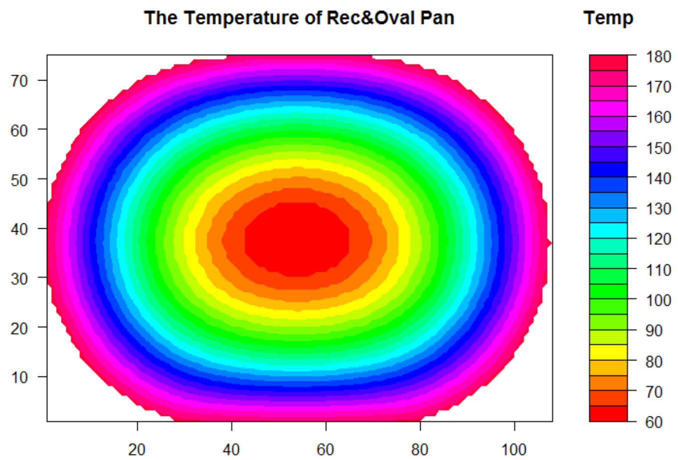


Figure 5.1.6

Oval pan

Set the length of the semi-major axis as a, then it should meet

$$\begin{cases} \frac{\pi a L}{2} = A \\ \frac{W}{L} = \frac{w}{l} \\ W > 0, L > 0 \end{cases} . \tag{5.1.11}$$

Solving for (5.1.11) we obtain

$$\begin{cases} L = \frac{2A}{\pi a} \\ W = \frac{2A}{\pi a}(\frac{w}{l}) \end{cases} . \tag{5.1.12}$$

Figure5.1.7 shows the schematic diagram of oval pan.

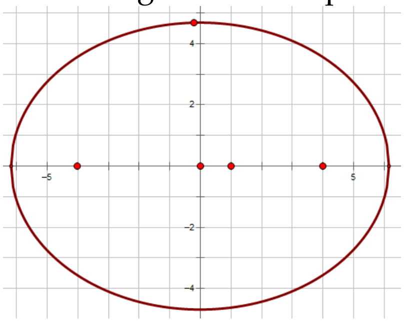


Figure 5.1.7 the schematic diagram of oval pan

Figure5.1.8 shows the heat distribution of the oval pan.

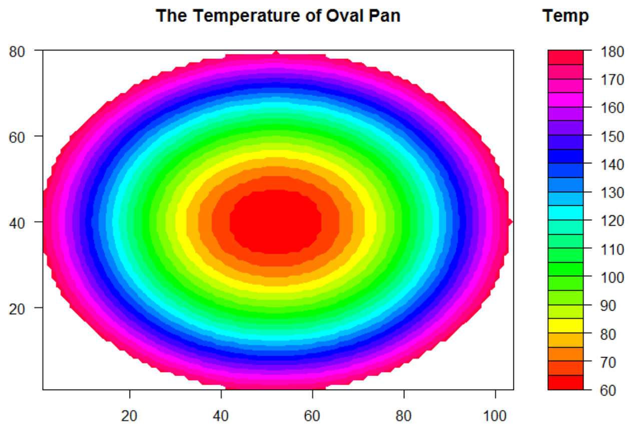


Figure 5.1.8

Round pan

Set the radius of the circle as r, then it should meet

$$\begin{cases} \pi r^2 = A \\ W = 2r \\ \frac{W}{L} = \frac{w}{l} \\ W > 0, L > 0 \end{cases} . \tag{5.1.13}$$

Solving for (5.1.13) we obtain

$$\begin{cases} W = 2\sqrt{\frac{A}{\pi}} \\ L = 2\sqrt{\frac{A}{\pi}}\left(\frac{l}{w}\right) \end{cases} . \tag{5.1.14}$$

Figure5.1.9 shows the schematic diagram of round pan.

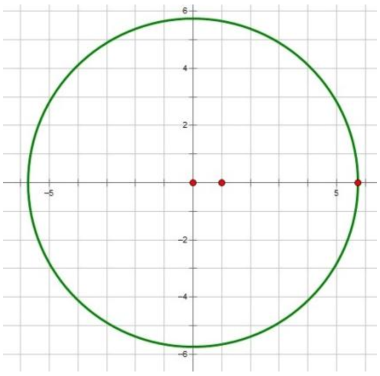


Figure 5.1.9 the schematic diagram of oval pan

Figure5.1.10 shows the heat distribution of the oval pan.

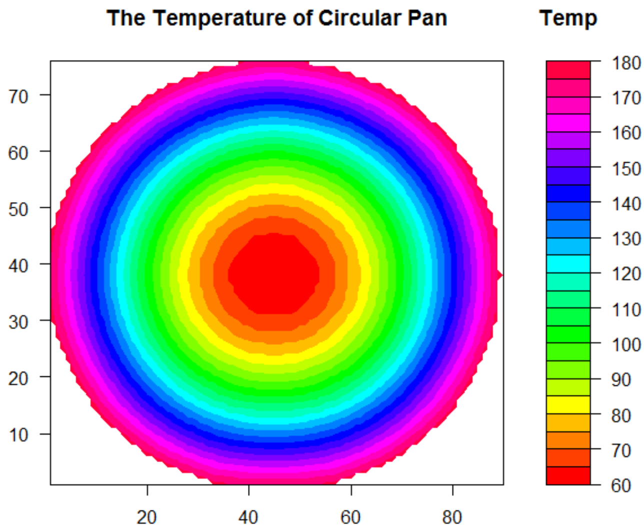


Figure 5.1.10

The temperature distribution of each point can be seen intuitively by the figures. However, in order to quantify the above results, we need to set a standard of temperature. Because the heat distribution of round pan is more uniform than the rectangle, and we find the temperature of the edges of the round one is approximately 177. Then, we set 177 as the standard temperature, which means we view the area whose temperature is higher than 177 is overcooked.

Calculate the number of the overcooked area of each type of pan. Next, calculate the proportion of the overcooked area in the entire region, and we define it as scorched rate. Then, we can obtain the shape that has the least scorched rate, which means it's the shape that has the most uniform distribution of heat.

In five different ratios of W/L (1:2, 3:4, 0.618:1, 1:1, 7:10), we calculate the scorched rate of different shapes by C++ Programming, and plot them as a line chart. (Figure 5.1.11-5.1.15)

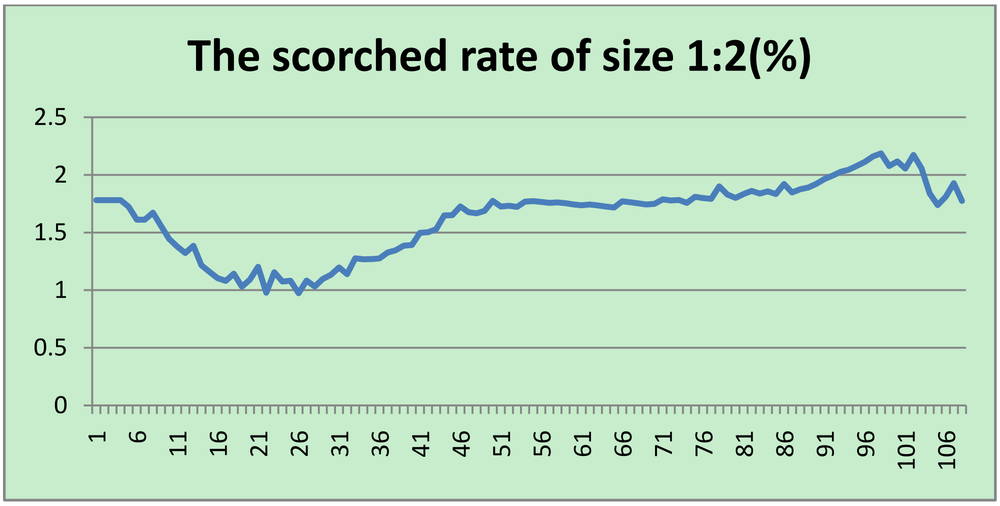


Figure 5.1.11

Note: Abscissa of 2-32 represents the different type of rounded rectangle. 33-78 represents the different type of oval plus rectangle. 79-107 represents the different type of oval. And the 1 represents rectangle, the 108 represents round.

Figure5.1.11 shows that 22 has the least scorched rate, which means the rounded rectangle is the best shape.

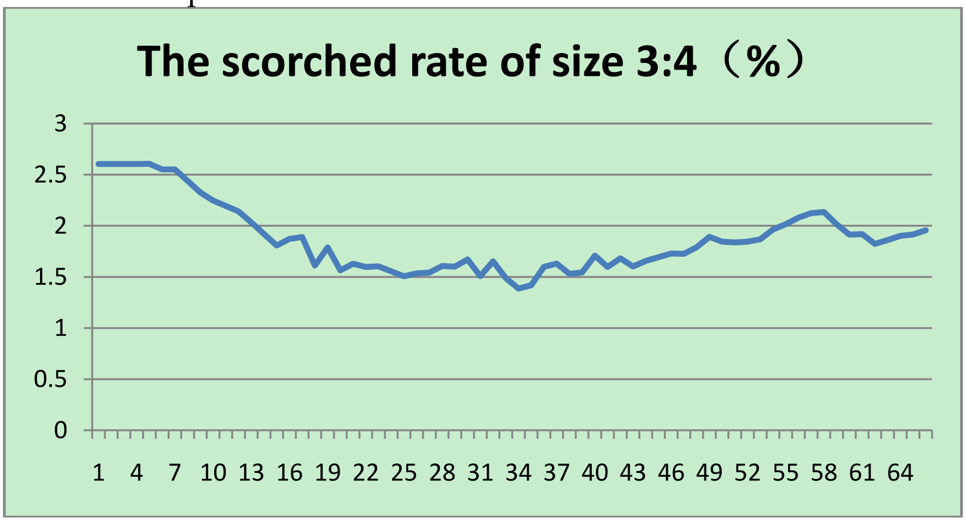


Figure 5.1.12

Note: Abscissa of 2-40 represents the different type of rounded rectangle. 41-57 represents the different type of oval plus rectangle. 58-65 represents the different type of oval. And the 1 represents rectangle, the 66 represents round.

Figure5.1.12 shows that 34 has the least scorched rate, which means the rounded rectangle is the best shape.

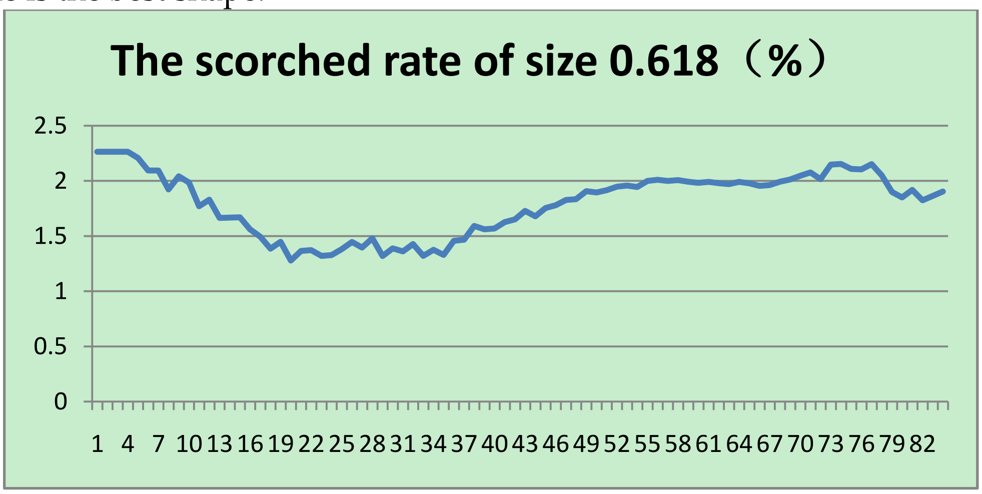


Figure 5.1.13

Note: Abscissa of 2-35 represents the different type of rounded rectangle. 36-66 represents the different type of oval plus rectangle. 67-83 represents the different type of oval. And the 1 represents rectangle, the 84 represents round.

Figure5.1.13 shows that 20 has the least scorched rate, which means the rounded rectangle is the best shape.

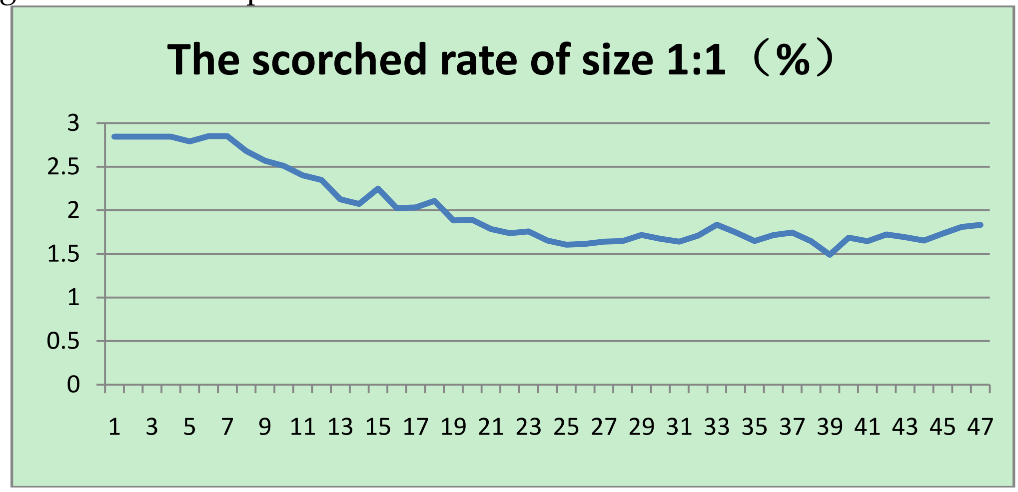


Figure 5.1.14

Note: Abscissa of 2-46 represents the different type of rounded rectangle. And the 1 represents rectangle, the 47 represents round.

Figure5.1.14 shows that 39 has the least scorched rate, which means the rounded rectangle is the best shape.

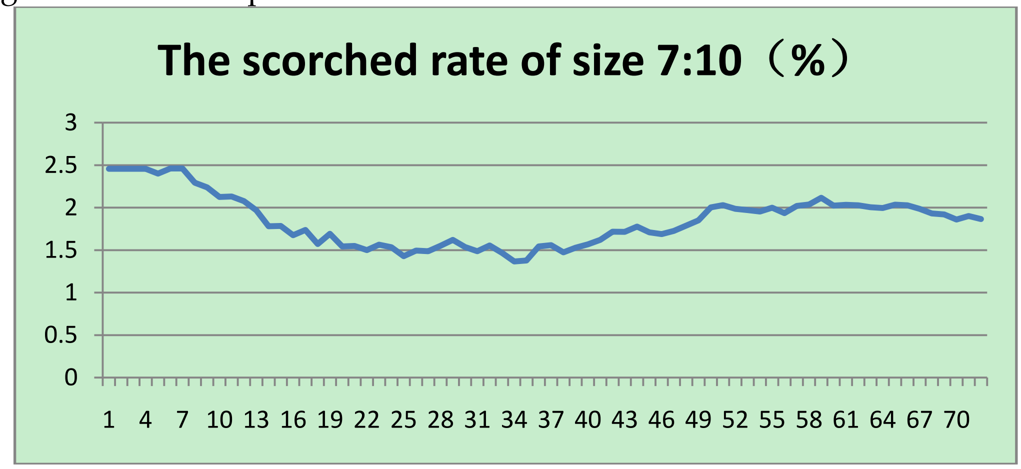


Figure 5.1.15

Note: Abscissa of 2-38 represents the different type of rounded rectangle. 39-60 represents the different type of oval plus rectangle. 61-71 represents the different type of oval. And the 1 represents rectangle, the 72 represents round.

Figure5.1.15 shows that 34 has the least scorched rate, which means the rounded rectangle is the best shape.

All the results show that the rounded rectangle is the best shape for distributing heat evenly.

5.2 Wasted rate model

5.2.1 The initial design of the pan

The oven is rectangular in shape, and the width to length ratio of W/L. So, we can set the width to length ratio as W/L to maximize number of pans that can fit in the oven (figure 5.2.1).

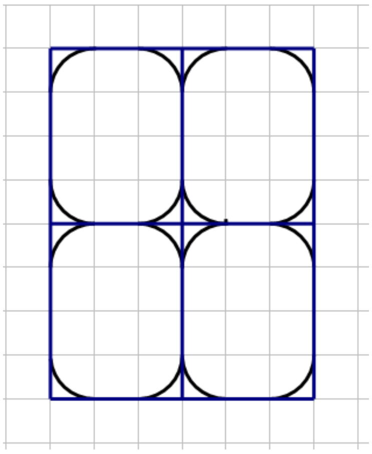


Figure5.21 the pan in the oven (a part of the whole)

Then, we change the shape of the pan as the analysis mentioned, which turn the corner of the rectangle to a sleek one. In order to keep the area of the pan as A, we just need to increase the length and the width by the same proportion. Figure5.2.1 shows the progress of the change in shape (we just use R2.15.1 to plot the upper part of the shape).

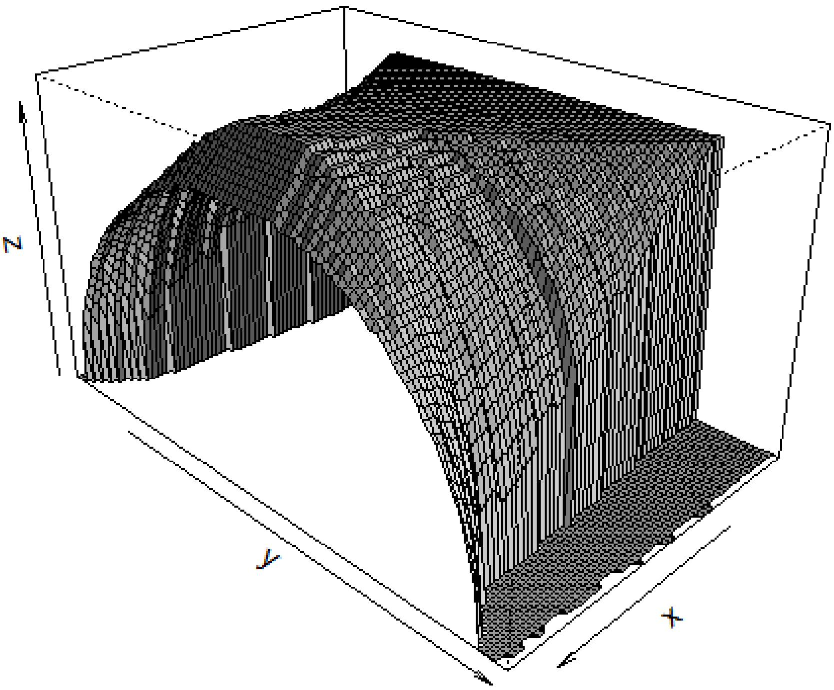


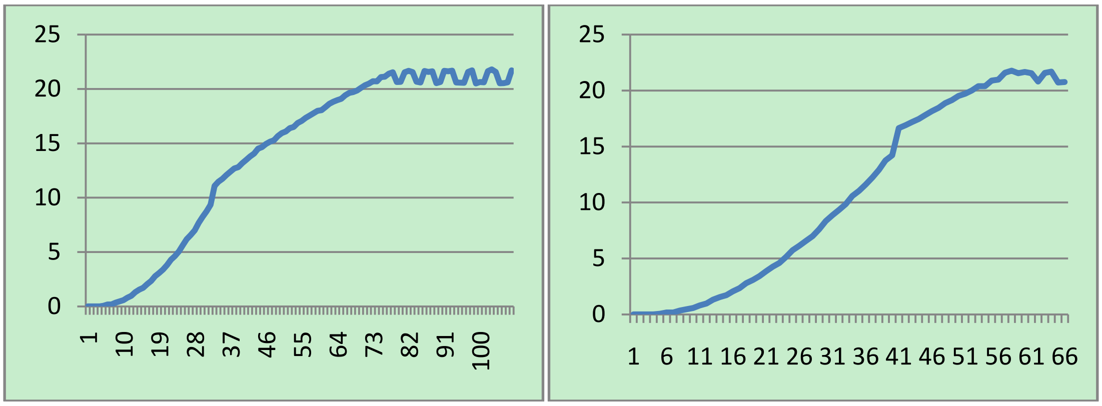
Figure5.2.1 the progress of the change in shape

5.2.2The calculating of wasted rate

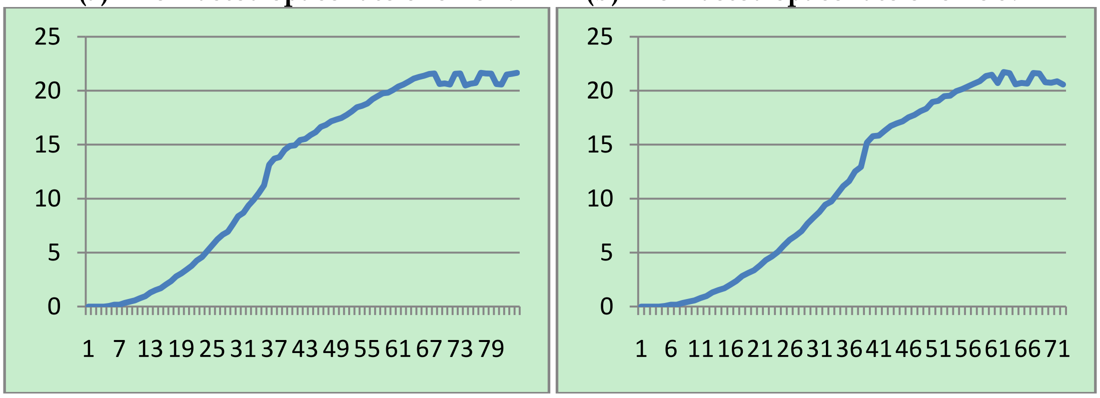
To calculate the wasted rate of the space, we first calculate the area of the rectangle that can exactly wrap the shape, and denote the area by S_{rec}. And we denote the wasted rate by R. Then we get the formula (5.2.1)

$$R = \frac{S_{rec} - A}{S_{rec}} \tag{5.2.1}$$

In five different ratios of W/L (1:2, 3:4, 0.618:1, 7:10), we calculate the wasted rate of different shapes by C++ Programming, and plot them as a line chart. (Figure 5.2.3)



(a) The wasted space rate of size 1:2 **(b)The wasted space rate of size 3:4**



(c) The wasted space rate of size 0.618:1 **(d) The wasted space rate of size 7:10**

Figure 5.2.3

All the figures show that with the shape of a rectangle into rounded, the wasted rate is increasing.

5.3 Evaluation model

5.3.1 Data reduction

To make the data dimensionless, we try to standardize them. However, we must make sure that the data is negative so that we can create a feasible objective function. Thus, we change the original formula into formula (1).

$$R_i' = \frac{R_i - R_{min}}{DR}, H_i' = \frac{H_i - H_{min}}{DH} \qquad (1)$$

5.3.2 Establishment of the model

After the data reduction, we make a scatterplot as figure5.3.1. The horizontal axis is the wasted rate and the longitudinal axis is the scorched rate.

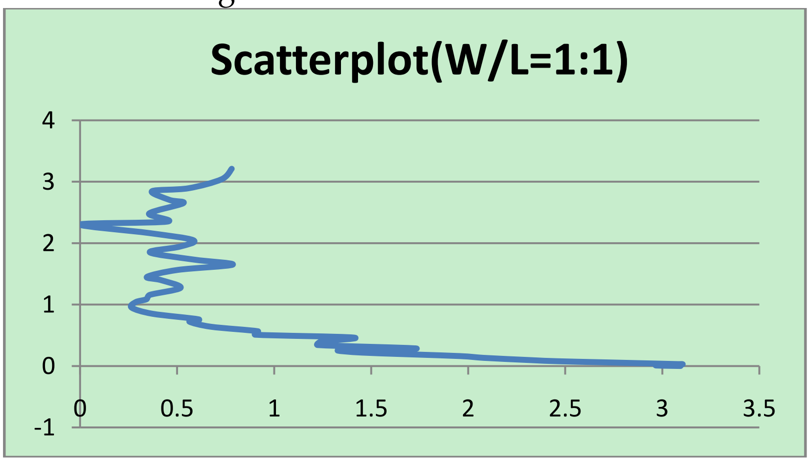


Figure 1 $R_i - H_i$ scatterplot (with line)

Then, we create the evaluation function:

$$z_{min} = pR + (1-p)H\,. \quad (5.3.2)$$

Solving for (5.3.2) we obtain

$$H = \frac{-p}{1-p}N + \frac{1}{1-p}z\ . \quad (5.3.3)$$

Take the linear programming method to get z_{min}. Make a line whose slope is $\frac{-p}{1-p}$ and move it to minimize the intercept, then the intercept is z_{min}.

However, it's impossible for us to get the graphical analytical solution, so that we find another way to get an approximate solution. Firstly, we make a scatterplot of the discrete value (Figure5.3.2).

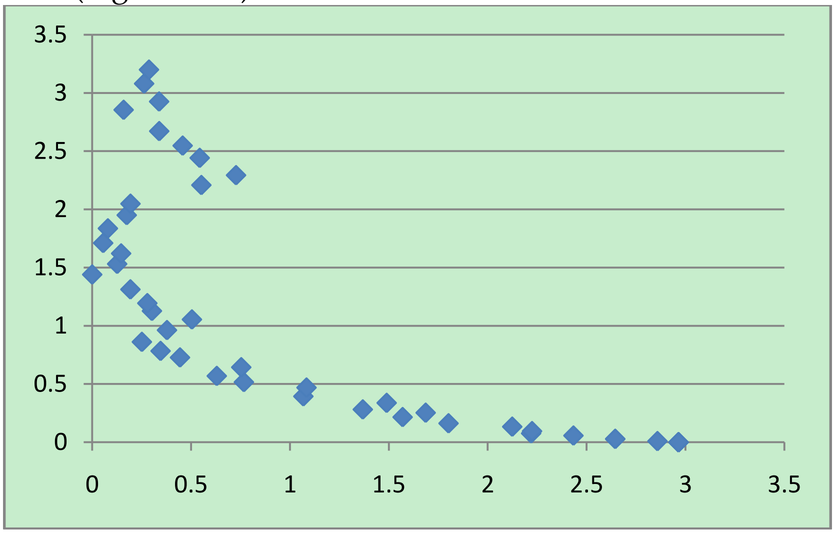


Figure 5.3.2 $R_i - H_i$ scatterplot

Then, calculate the smallest convex hull that is able to wrap them. The progress is as follows.

Step 1: Obtain the point Q1 whose abscissa is the smallest, and view it as the current point Q.

Step 2: Make lines between Q and every other point. Then, pick out the line that has the smallest slope, and denote the point as Q'.

Step 3: If Q' has the smallest ordinate, then end, otherwise, view it as the current point Q and back to step 2.

After the above steps, we get a polyline which is a part of the convex hull. And the line of formula (5.3.3) must intersect with the polyline. Then, we can obtain the shape of pan corresponding to different P by following progress.

1. Calculate the slope(k_i) of every line segment(S_i) of the polyline.
2. Since we have got k_i, we can calculate the value of P_i by the formula $P_i = \frac{k_i}{k_i - 1}$.
3. Obtain the P_i of the two segments that are connected with the Q. If P belongs to the $[P_i, P_j]$, then the best shape for P is the shape of Q.

5.3.3 Model solution

Since we have the conclusion that the rounded rectangle is the best shape, we just need to calculate the radius of the round to create the pan. We assume there are five different kind of ovens whose size are 0.1:1, 0.2:1, 0.5:1, 0.618:1, 1:1. Then, choose the best radius for each of them by C++ programming. That is, first we calculate the R_i and H_i corresponding to different radius of the rounded rectangle. Next, we standardize the R_i and H_i and plot a scatterplot of the dimensionless data. Use the scatterplot to make a smallest convex hull. At last, we get the best radius for different P in five different ovens. Table5.3.1 shows the summary of results.

Table5.3.1 the rounded rectangle pan for five ovens in different p

W/L	The range of P	radius	Scorched rate (%)	Wasted rate (%)
1:10	$[0.5312, 1]$	0	0.2312	0
	$[0.4013, 0.5312)$	5	0.1447	0.0867
	$[0.119, 0.4013)$	8	0.1159	0.2312
	$[0, 0.119)$	16	0	1.2044
1:5	$[0.6439, 1]$	0	0.6702	0
	$[0.2642, 0.6439)$	5	0.5833	0.0874
	$[0.1604, 0.2642)$	8	0.5138	0.2278
	$[0.0868, 0.1604)$	12	0.4013	0.6550
	$[0, 0.0868)$	16	0.3733	1.1917
1:2	$[0.6973, 1]$	0	2.3375	0
	$[0.4517, 0.6973)$	5	2.2529	0.0866
	$[0.4303, 0.4517)$	8	2.1404	0.2307
	$[0.2857, 0.4303)$	11	2.0300	0.4906
	$[0.2727, 0.2857)$	12	1.9686	0.6519
	$[0.2461, 0.2727)$	14	1.9154	0.8503
	$[0.1871, 0.2461)$	16	1.8359	1.1905
	$[0.1709, 0.1871)$	19	1.7202	1.7188
	$[0.1011, 0.1709)$	22	1.6450	2.3669
	$[0.0823, 0.1011)$	23	1.6200	2.6021
	$[0, 0.0823)$	29	1.5536	4.3481
0.618:1	$[0.9219, 1]$	0	2.9855	0
	$[0.5357, 0.9219)$	5	2.9011	0.0866
	$[0.2932, 0.5357)$	9	2.7616	0.2899
	$[0.2321, 0.2932)$	10	2.7066	0.4058
	$[0.2180, 0.2321)$	15	2.475	1.0254
	$[0.1889, 0.2180)$	16	2.4216	1.1963
	$[0.1551, 0.1889)$	18	2.3150	1.5666
	$[0.1124, 0.1551)$	22	2.1449	2.3769
	$[0.0823, 0.1124)$	27	2.0493	3.6434
	$[0, 0.0823)$	48	1.9908	11.5816
1:1	$[0.9518, 1]$	0	3.7633	0
	$[0.4265, 0.9518)$	5	3.6803	0.0862
	$[0.3048, 0.4266)$	10	3.4324	0.4022
	$[0.1946, 0.3048)$	14	3.1907	0.8474
	$[0.0981, 0.1946)$	21	2.8594	2.2032
	$[0.0670, 0.0981)$	25	2.7260	3.1944
	$[0.0577, 0.0670)$	32	2.5654	5.2182
	$[0, 0.0577)$	46	2.5492	10.624

Note: we assume the area of the pan is 14000, and the value of radius is calculated at the condition of A=14000.

In order to find the connection between W/L and the shape of pan, we calculate the R_i and H_i of the best shape for nine kinds of oven and the different weight P. And we plot them in a scatterplot with different colors (figure5.3.3).

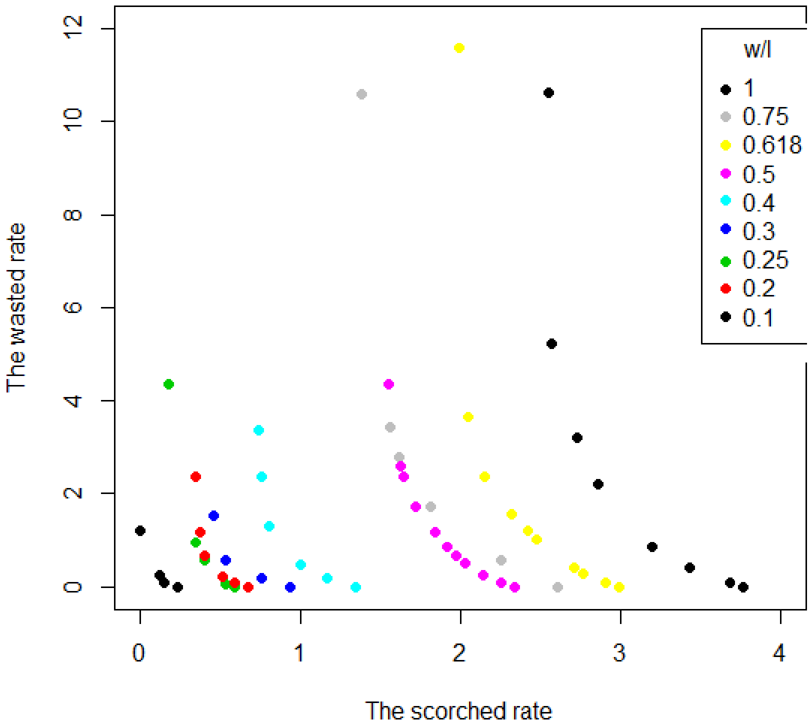


Figure 5.5.3 $R_i - H_i$ scatterplot

Since the lower R_i and H_i are, the better the shape is. In figure3, if point Q_i is in the bottom left of Q_j, then Q_i is better than Q_j. Thus, we can get the conclusion that the smaller the W/L is the better the shape is.

6. Final design

After we solved the models, we find that the smaller W/L is the better the shape is. Thus, we obtain the final progress to get a best pan.

First, assume the area of the oven that can put the pan is S, the area of the pan is A, and the width to length ratio is W/L. Then, we can calculate the length and width of the oven by formula $L = \sqrt{\frac{S}{W/L}}, W = \frac{S}{L}$. The maximum number $N = \left\lfloor \frac{S}{A} \right\rfloor$.

In the condition of the maximum number is N, we can know that the maximum length of the pan is L , the maximum width is [W/N], and the minimum width is [A/L].

P=0 represents the more number of pans the better, thus set width as W/N is the best shape.

P=1 represents the shape only depends on the scorched rate. After calculating, we obtain the best radius of the rounded rectangle pan is one third of width, and $r = \frac{3L-\sqrt{9L^2-4A(4-\pi)}}{2(4-\pi)}$.

If 0<P<1, we can use the C++ programming that we have created to solve the best shape out.

7. Analysis of the Models

7.1 Sensitivity analysis

In the model solutions above, we discretize the heating area into a grid of 100×100 in the condition of A=14000. To find whether the discretize accuracy has an influence on the result, we double the area of the pan and discretize the heating area into a grid of 141×141, then solve the result out. We find that the rounded rectangle is still the best shape. Figure 7.1.1 shows the result of two kind of accuracy.

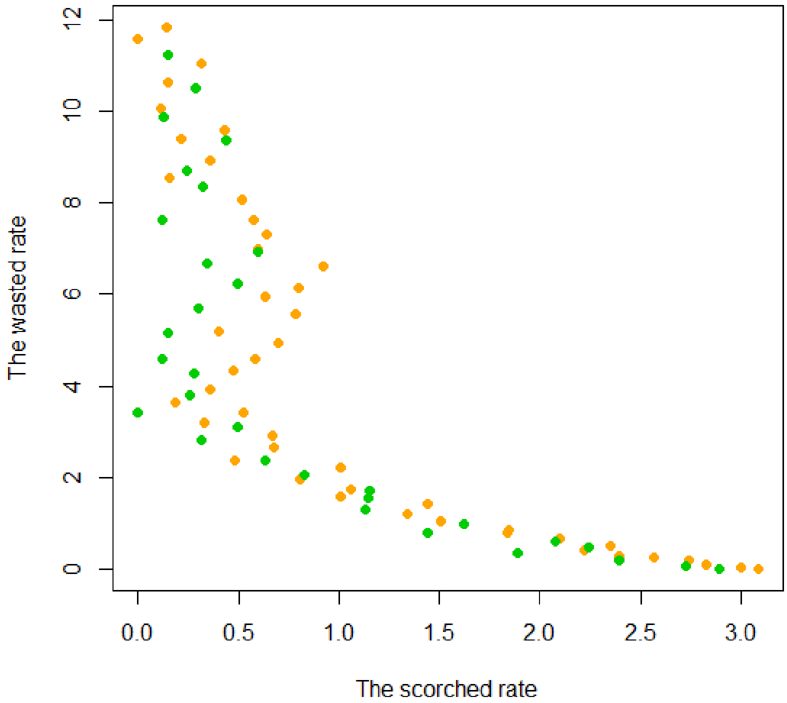


Figure 7.1.1 the result of two kind of accuracy

Note: the green one is the result before improving the accuracy, and the yellow one is the result after improving the accuracy.

As the figure7.1.1 shows, the discretize accuracy only has a small influence on the result.

7.2 Strengths

The fundamental strengths of our model are its robustness and flexibility. All of the data is fully parameterized, so the model can be applied to illustrate how the results vary with different values of W/L, p, and many other factors. It was fast but also had the sensitivity we desired.

In addition, our model is well matched up with real life. It produced table5.3.1 that show how the results vary with different intervals of w/l and p, which is convenient for users to choose and use.

7.3 Weakness

Because of the limited time, the models have a low precision. Our results lack clear illustrative power, and data manipulated by a computer program cannot achieve the same effect as analytical solution.

Reference

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【3】 Emmanuel Paradis. R for beginners[M].Beijing: China Machine PRESS,2011.
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Appendix

（1）The area of the pan is A, and the width to length ratio of the oven is W/L. Below is the C++ code for calculating the wasted rate and scorched rate of the best shape .

```
#include <iostream>
#include <fstream>
#include <cmath>
#define Pi 3.1415926536
using namespace std;
double temp[1002][1002][2];
double wlratio = 0.618,A = 7000;
int w = 100,l;
double initemp = 20,maxtemp = 180, ripetemp = 60, overtemp = 177;
double F = 0.22;
int ans[1002],i,j;
double dist(int a,int b,double c,double d){//count the dist
	return sqrt(((a-c)*(a-c)+(b-d)*(b-d)));
}
bool check(int i,int j,int type,int a){//check if (i,j)is in the shape(a)
	switch (type){
		case 1:
			if (i<=a && j<=a || i>w-a && j<=a || i<=a && j>l-a || i>w-a && j>l-a){
				if (dist(i,j,a,a)<a || dist(i,j,a,l-a+1)<a ||dist(i,j,w-a+1,a)<a||dist(i,j,w-a+1,l-
	a+1)<a) return true;
				return false;
			}
			return true;
			break;
		case 2:
			if (i<=a || i>w-a){
				double c = sqrt((a*a-l*l/4)*1.0);
				if (dist(i,j,a-c,l/2)+dist(i,j,a+c,l/2)>2*a && dist(i,j,w-a-c,l/2)+dist(i,j,w-
	a+c,l/2)>2*a) return false;
				return true;
			}
			return true;
		case 3:
```

```
            double c = sqrt((a*a-l*l/4)*1.0);
            if (dist(i,j,a-c,l/2)+dist(i,j,a+c,l/2)>2*a) return false;
            return true;
    }
}
double getans(int type,int a){

    w = static_cast<int>(sqrt((A+(4-Pi)*a*a)/wlratio));
        l = w*wlratio;
        for (int i=0;i<=w+1;i++)
        for (int j=0;j<=l+1;j++){
            temp[i][j][0] = temp[i][j][1] = maxtemp;
        }
    for (int i=1;i<=w;i++)
        for (int j=1;j<=l;j++){
            temp[i][j][0] = check(i,j,type,a)?initemp:maxtemp;
        }

    int t=0;
    while (true){
        int number = 0;
        for (int i=1;i<=w;i++)
            for (int j=1;j<=l;j++)if (check(i,j,type,a)){
                temp[i][j][1-t] = F*(temp[i-1][j][t] + temp[i+1][j][t] + temp[i][j-1][t] +
    temp[i][j+1][t])+(1-4*F)*temp[i][j][t];
                if (temp[i][j][1-t] <ripetemp) number ++;
            }
        if (number == 0) break;
        t=1-t;
    }

    int number = 0,sum = 0;
    for (int i=1;i<=w;i++){
        for (int j=1;j<=l;j++){
            if (check(i,j,type,a)) sum++;
        }
    }
    for (int i=1;i<=w;i++){
        for (int j=1;j<=l;j++){
            if (check(i,j,type,a)&&temp[i][j][1-t]>overtemp) number++;
        }
    }
    return number*100.0/sum;

}
double getrate(int type,int a){
    w = static_cast<int>(sqrt((A+(4-Pi)*a*a)/wlratio));
        l = w*wlratio;
    int sum = 0,x = 200,y = 200,xx = 0,yy = 0;
    for (int i=1;i<=w;i++){
        for (int j=1;j<=l;j++){
            if (check(i,j,type,a)) {
                sum++;
```

```
                        if (i<x) x = i;
                        if (i>xx) xx = i;
                        if (j<y) y = j;
                        if (j>yy) yy = j;
                }}
        }
        return 100 - sum * 100.0/((xx-x+1)*(yy-y+1));

}
int main(){
        if (wlratio>1) wlratio = 1/wlratio;
        ofstream output("result.txt");

        int type=1;
        for (int a=0;a<=l/2;a++){
                output<<type<<' '<<a<<' '<<getans(type,a)<<' '<<getrate(type,a)<<endl;
        }

        type = 2;
        for (int a=l/2+1;a<=w/2;a++){
                output<<type<<' '<<a<<' '<<getans(type,a)<<' '<<getrate(type,a)<<endl;
                }
        type = 3;
        for (int a=w/2;a>sqrt(A/Pi);a--){
                output<<type<<' '<<a<<' '<<getans(type,a)<<' '<<getrate(type,a)<<endl;
        }
        return 0;
}
```

(2) The area of the pan is A, print the width to length ratio and the radius of the pan to calculate the area of the rectangle that can exactly wrap the shape. Here is the code.

```
#include <iostream>
#include <fstream>
using namespace std;
int main(){
        while (true){
                int a,A=7000;
                double c,Pi=3.1415926536;
                double w,l;
                while (cin>>c>>a){
                        w = (sqrt((A+(4-Pi)*a*a)/c));
                        l = w*c;
                        cout<<w<<' '<<l<<endl;
                }
        }
}
```

(3) The code to calculate the smallest convex hull.

```
#include <iostream>
#include <fstream>
using namespace std;
```

```
int main(){
	ifstream input("resultl.txt");
	ofstream output("result.txt");
	double a[1000],b[1000];

	int n=0,mina=0,minb=0;
	while (!input.eof()){
		input>>a[n]>>a[n]>>a[n]>>b[n];

		if (a[n]<=a[mina]) mina = n;
		if (b[n]<b[minb]) minb = n;
		n++;
	}
	output<<a[mina]<<' '<<b[mina]<<endl;
	int p=mina;
	while (p!=minb) {
		double k,kmin=1000000; int kk;
		for (int i=0;i<n;i++) if (a[i]>a[p]){
			if (abs(a[i]-a[p])>0.000001) k=(b[i]-b[p])/(a[i]-a[p]);
			if (k<kmin) { kmin = k; kk = i;};
		}
		p=kk;
		output<<a[p]<<' '<<b[p]<<endl;
	}
	return 0;
}
```

(4) The code to calculate shape of different weight P.

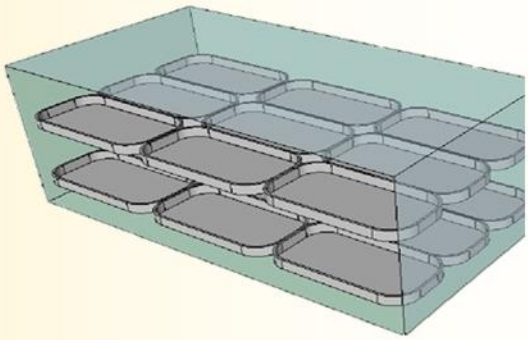
```
#include <iostream>
#include <fstream>
using namespace std;
int main(){
	ofstream output("outgold.txt");
double a[200],b[200],c[200],p;
int n,i;
cin>>n;
	for (i=0;i<n;i++) cin>>a[i]>>b[i];

	int pre = -1;
	for (p=0;p<1;p+=0.00001){
		int min = 0;
		for (i=0;i<n;i++){
			c[i] = a[i]*p+b[i]*(1-p);
			if (c[i]<c[min]) min = i;
			}
		if (min!=pre){
			output<<p<<' '<<min<<endl;
			pre= min;
		}
	}
	return 0;
	}
```

Never worry about your baking

—Gourmet Gospel

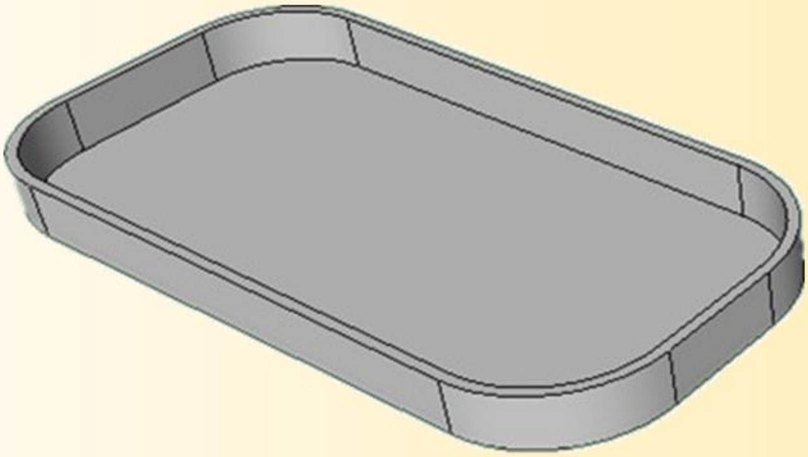
The creation of Brownie, as many other delicious foods, is a graceful accident. The biggest challenge before we enjoy the cute Brownies, which depart us from Brownie Pans, is timing. When baking in a rectangular pan, the cake gets overcooked at the corners, while round pans are too unwieldy, which take too much place in the oven.

Therefore, the emergence of the Ultimate Brownie Pan is gourmets Gospel. The shape of the pan is a rounded rectangle, which makes the heat distributed evenly, and, in the other hand, it can be regularly placed in the oven, which means it will get full use of the space.



"We synthesized the utilization of the space and the distribution of the heat of each shape of the pan," said the chief designer of the Ultimate Brownie Pan, "and the rounded rectangle is the optimal solution."

Why we choose the rounded rectangle as the optimal solution?



To find a pan which can distribute heat most evenly, we take rectangle, rounded rectangle, oval plus rectangle, oval and round into consideration. We find that the rounded rectangle is the best shape which makes the cake won't get overcooked.

To maximize the number of pans that can fit in the oven, which can be regarded as making best use of the space, the rectangle pan should be the best choice. However, the corners of the cake are overcooked easily by the rectangular one, which gravely affects the tasty of the cake.

Take both conditions into account, we obtain the following results:

If we know the width to length ratio of the oven, then choose the pan that has the same length to width ratio with the oven, and the radius is one third of the width.

If we know the area of the oven and the area of the pan, then choose the pan that has the longest length.